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RECIPROCAL GUTMAN INDEX OF PRODUCTS OF COMPLETE GRAPHS

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ABSTRACT

The Reciprocal Gutman Index (RGut), defined for a connected graph as vertex degree weighted sum of the reciprocal distances, i.e, $RGut(G) = \sum_{u,v \in V(G)} \frac{d_G(u)d_G(v)}{d_G(u,v)}$. The RGut is a weighted version of the Harary index, just as the Gutman index is a weighted version of the Wiener index. In this paper, we present exact expression for the reciprocal Gutman index of Cartesian product and the wreath product of complete graphs with odd vertices interms of other graphs invariants including the reciprocal degree distance, Harary index, the first Zagreb index and the second Zagreb index.

KEYWORDS: Reciprocal Gutman index, Reciprocal degree distance, Cartesian Product, Wreath Product, Harary index, first and second Zagreb indices.

INTRODUCTION

In this paper, all the graphs are considered as compete graphs with odd vertices. For vertices $u, v \in V(G)$, the distance between u and v in G, denoted by $d_G(u,v)$ is the length of the shortest (u,v) – path in G and let $d_G(v)$ be the degree of a vertex $v \in V(G)$. For two complete graphs $G_1$and G_2 their Cartesian product denoted by $G_1 \blacksquare G_2$ has the vertex set $V(G_1 \blacksquare G_2) = V(G_1) \times V(G_2)$ and $(u,x)(v,y)$ is an edge of $G_1 \blacksquare G_2$if $u = v$ and $xy \in E(G_2)$ or $uv \in E(G_1)$ and $x = y$. The wreath product of the graphs $G_1$and G_2 denoted by $G_1 {}^\circ G_2$ is the graph with vertex set $V(G_1) \times V(G_2)$ and $(u,x)(v,y)$ is an edge of $G_1 {}^\circ G_2$if $u = v$ and $xy \in E(G_2)$ or $uv \in E(G_1)$. A topological index of a graph is a real number related to the graph; it does not depand on labeling or pictorial representation of a graph. In theoretical chemistry, molecular structure descriptors(also called topological indices) are used for modeling physicochemical, pharmacologic, toxicological, biological and other properties of chemical compounds[6]. There exist several types of such indices, especially those based on vertex and edge distances. One of the most intensively studied topological indices is the Wiener index; for other related topological indices see [17].

Let Gbe a connected graph. Then the Wiener index of $G$is defined as $W(G) = \frac{1}{2}\sum_{u,v \in V(G)} d_G(u,v)$ with the summation going over all pairs of distinct vertices of G. Similarly the Harary index of G is defined as $(G) = \frac{1}{2}\sum_{u,v \in V(G)} \frac{1}{d_G(u,v)}$. Dobrynin and Kochetova [3] and Gutman[8] independently proposed a vertex degree-weighted version of wiener index called degree distance or scholtz molecular toplogical index, which is defind for a connected graph G as $DD(G) = \frac{1}{2}\sum_{u,v \in (VG)} d_G(u,v)[d_G(u) + d_G(v)]$, where $d_G(u)$ is the degree of the vertex u in G. Note that the degree distance is a degree- weight version of the wiener index. Hua and Zhang [13] introduced a new graph invariant named reciprocal degree distance which can be seen as a degree –weight version of Harary index, that is $RDD(G) = \frac{1}{2}\sum_{u,v \in (VG)} \frac{[d_G(u) + d_G(v)]}{d_G(u,v)}$. Reciprocal Degree Distance of some graph operations has been obtained in [15] . In [8] , Gutman defined the Schultz index of the second kind which is now dnown as the Gutman index . The Gutman index of G denoted by $Gut(G)$, is defined as $Gut(G) = \frac{1}{2}\sum_{u,v \in (VG)} d_G(u,v)[d_G(u)d_G(v)]$ with the summation runs over all the pairs of vertices of G. The Gutman indices of product graphs have been obtained in [14].

There are some topological indices based on degrees such as the first and second Zagreb indices of molecular graphs. The first and second kinds of zgreb indices are introduced in [17]. The first Zagreb index $M_1(G)$ and the second

Zagreb index $M_2(G)$ of a graph G are defined as, $M_1(G) = \sum_{uv\in E(G)}[d_G(u) + d_G(v)] = \sum_{v\in V(G)} d_G^2(v)$and $M_2(G) = \sum_{uv\in E(G)}[d_G(u)d_G(v)]$ The Multiplicative Zagreb indices of graph operations have been obtained in [18].

In this paper, we found a new graph invariant named reciprocal Gutman index, which can be seen as a degree weight version of Harary index that is $RGut(G) = \frac{1}{2}\sum_{u,v\in V(G)} \frac{d_G(u)d_G(v)}{d_G(u,v)}$

And we obtain the reciprocal Gutman indices of Cartesian and wreath products of complete graphs with odd vertices.

GUTMAN INDEX OF CARTESIAN PRODUCT OF COMPLETE GRAPHS

In this section, we find the reciprocal Gutman index of the Cartesian product $G_1 \blacksquare G_2$ of the complete graphs G_1 and $G_2$with odd vertices. Let $V(G_1) = \{u_0, u_1, \cdots, u_{n_1-1}\}, V(G_2) = \{v_0, v_1, \cdots, v_{n_2-1}\}$ and let w_{ij} be the vertex (u_i, v_j) of $G_1 \blacksquare G_2$.

The following lemma follows from the definition of the Cartesian product of the two graphs.

Lemma – 2.1
Let $G_1$and G_2 be two complete graphs. Let $w_{ij} = (u_i, v_j)$ and $w_{pq} = (u_p, v_q)$ be in $V(G_1 \blacksquare G_2)$. Then $d_{G_1 \blacksquare G_2}(w_{ij}, w_{pq}) = 2d_{G_1}(u_i, u_p)$ and $d_{G_1 \blacksquare G_2}(w_{ij}) = d_{G_1}(u_i) + d_{G_2}(v_j)$.

Theorem – 2.2
If $G_1$and G_2 are two complete graphs with odd vertices$|V(G_1)| = n_1$and $|V(G_2)| = n_2$, where $n_1, n_2 \geq 3$ then

$$RGut(G_1 \blacksquare G_2) = \frac{1}{2}[H(G_1)M_1(G_2) + 4e(G_2)RDD(G_1) + 2n_2RGut(G_1) + 2H(G_2)M_1(G_1) + 4e(G_1)RDD(G_2) + 2n_1RGut(G_2) + n_2(n_2 - 1)RGut(G_1) + 4n_1(n_1 - 1)(n_2 - 1)e(G_2) + 4H(G_1)e(G_2)^2]$$

where $RGut(G), H(G), M_1(G)$ and $RDD(G)$ denote the reciprocal Gutman index, the Harary index , the first Zagreb index and the reciprocal degree distance of G respectively.

Proof

Let $G = G_1 \blacksquare G_2$. Then

$$RGut(G) = \frac{1}{2}\sum_{w_{ij},w_{pq}\in V(G)} \frac{d_G(w_{ij})d_G(w_{pq})}{d_G(w_{ij},w_{pq})} =$$

$$\frac{1}{2}\left\{\sum_{j=0}^{n_2-1}\sum_{i,p=0,i\neq p}^{n_1-1} \frac{d_G(w_{ij})d_G(w_{pj})}{d_G(w_{ij},w_{pj})} + \sum_{i=0}^{n_1-1}\sum_{j,q=0,j\neq q}^{n_2-1} \frac{d_G(w_{ij})d_G(w_{iq})}{d_G(w_{ij},w_{iq})} + \sum_{j,q=0,j\neq q}^{n_2-1}\sum_{i,p=0,i\neq p}^{n_1-1} \frac{d_G(w_{ij})d_G(w_{pq})}{d_G(w_{ij},w_{pq})}\right\}$$

$$= \frac{1}{2}(A + B + C) \qquad (2.1)$$

where A, B, C are the sums of the above terms in order. We shall calculate A, B, C of (2.1) separately.
First we calculate,

$$A = \sum_{j=0}^{n_2-1}\sum_{i,p=0,i\neq p}^{n_1-1} \frac{d_G(w_{ij})d_G(w_{pj})}{d_G(w_{ij},w_{pj})}$$

For this we firsr compute

$$A' = \sum_{i,p=0,i\neq p}^{n_1-1} \frac{d_G(w_{ij})d_G(w_{pj})}{d_G(w_{ij},w_{pj})}$$

$$= \sum_{i,p=0,i\neq p}^{n_1-1} \frac{[d_{G_1}(u_i)+d_{G_2}(v_j)][d_{G_1}(u_p)+d_{G_2}(v_j)]}{d_{G_1}(u_i,u_p)}$$

by the Lemma – 2.1

$$= \sum_{i,p=0,i\neq p}^{n_1-1} \frac{[d_{G_1}(u_i)d_{G_1}(u_p)]}{d_{G_1}(u_i,u_p)} + d_{G_2}(v_j)\sum_{i,p=0,i\neq p}^{n_1-1} \frac{[d_{G_1}(u_i)+d_{G_1}(u_p)]}{d_{G_1}(u_i,u_p)} + d_{G_2}^2(v_j)\sum_{i,p=0,i\neq p}^{n_1-1} \frac{1}{d_{G_1}(u_i,u_p)}$$

$$= 2RGut(G_1) + 2d_{G_2}(v_j)RDD(G_1) + 2d_{G_2}^2(v_j)H(G_1)$$

by the definitions of reciprocal Gutman index, reciprocal degree distance and Harary index of a graph.

$$\therefore A = \sum_{j=0}^{n_2-1}\left[2RGut(G_1) + 2d_{G_2}(v_j)RDD(G_1) + 2d_{G_2}^2(v_j)H(G_1)\right]$$
$$= 2n_2RGut(G_1) + 4e(G_2)RDD(G_1) + 2H(G_1)M_1(G_2) \qquad (2.2)$$

Next we calculate

$$B = \sum_{i=0}^{n_1-1}\sum_{j,q=0,j\neq q}^{n_2-1}\frac{d_G(w_{ij})d_G(w_{iq})}{d_G(w_{ij},w_{iq})}$$

For this we compute

$$B' = \sum_{j,q=0,j\neq q}^{n_2-1}\frac{d_G(w_{ij})d_G(w_{iq})}{d_G(w_{ij},w_{iq})}$$
$$= \sum_{j,q=0,j\neq q}^{n_2-1}\frac{[d_{G_1}(u_i)+d_{G_2}(v_j)][d_{G_1}(u_i)+d_{G_2}(v_q)]}{d_{G_2}(v_j,v_q)} \qquad \text{by lemma – 2.1}$$
$$= \sum_{j,q=0,j\neq q}^{n_2-1}\frac{[d_{G_2}(v_j)d_{G_2}(v_q)]}{d_{G_2}(v_j,v_q)} + d_{G_1}(u_i)\sum_{j,q=0,j\neq q}^{n_2-1}\frac{[d_{G_2}(v_j)+d_{G_2}(v_q)]}{d_{G_2}(v_j,v_q)} + d_{G_1}{}^2(u_i)\sum_{j,q=0,j\neq q}^{n_2-1}\frac{1}{d_{G_2}(v_j,v_q)}$$
$$= 2RGut(G_2) + 2d_{G_1}(u_i)RDD(G_2) + 2d_{G_1}^2(u_i)H(G_2)$$

by the definitions of reciprocal Gutman index, reciprocal degree distance and Harary index of a graph.

$$\therefore B = \sum_{i=0}^{n_1-1}\left[2RGut(G_2) + 2d_{G_1}(u_i)RDD(G_2) + 2d_{G_1}^2(u_i)H(G_2)\right]$$
$$= 2n_1RGut(G_2) + 4e(G_1)RDD(G_2) + 2H(G_2)M_1(G_1) \qquad (2.3)$$

Next we compute

$$C = \sum_{j,q=0,j\neq q}^{n_2-1}\sum_{i,p=0,i\neq p}^{n_1-1}\frac{d_G(w_{ij})d_G(w_{pq})}{d_G(w_{ij},w_{pq})}$$

For this we compute

$$C' = \sum_{i,p=0,i\neq p}^{n_1-1}\frac{d_G(w_{ij})d_G(w_{pq})}{d_G(w_{ij},w_{pq})}$$
$$= \sum_{i,p=0,i\neq p}^{n_1-1}\frac{[d_{G_1}(u_i)+d_{G_2}(v_j)][d_{G_1}(u_p)+d_{G_2}(v_q)]}{2d_{G_1}(u_i,u_p)} \qquad \text{by lemma – 2.1}$$
$$= \sum_{i,p=0,i\neq p}^{n_1-1}\frac{[d_{G_1}(u_i)d_{G_1}(u_p)]}{2d_{G_1}(u_i,u_p)} + \sum_{i,p=0,i\neq p}^{n_1-1}\frac{[d_{G_1}(u_i)d_{G_2}(v_q)]}{2d_{G_1}(u_i,u_p)} + \sum_{i,p=0,i\neq p}^{n_1-1}\frac{d_{G_2}(v_j)d_{G_1}(u_p)}{2d_{G_1}(u_i,u_p)} + \sum_{i,p=0,i\neq p}^{n_1-1}\frac{d_{G_2}(v_j)d_{G_2}(v_q)}{2d_{G_1}(u_i,u_p)}$$
$$= RGut(G_1) + d_{G_2}(v_j)n_1(n_1-1) + d_{G_2}(v_q)n_1(n_1-1) + H(G_1)d_{G_2}(v_j)d_{G_2}(v_q)$$

by the definitions of reciprocal Gutman index and Harary index of a graph.

$$\therefore C = \sum_{j,q=0,j\neq q}^{n_2-1}\left[RGut(G_1) + d_{G_2}(v_j)n_1(n_1-1) + d_{G_2}(v_q)n_1(n_1-1) + H(G_1)d_{G_2}(v_j)d_{G_2}(v_q)\right]$$
$$= n_2(n_2-1)RGut(G_1) + 2e(G_2)(n_2-1)n_1(n_1-1) + 2e(G_2)(n_2-1)n_1(n_1-1) + H(G_1)(4e(G_2)^2 - M_1(G_2)), \qquad (2.4)$$

since $\sum_{j,q=0,j\neq q}^{n_2-1} d_{G_2}(v_j)d_{G_2}(v_q) = 4e(G_2)^2 - M_1(G_2)$

Using $(2.2), (2.3), (2.4)$ in (2.1), we get

$$RGut(G) = \frac{1}{2}[H(G_1)M_1(G_2) + 4e(G_2)RDD(G_1) + 2n_2RGut(G_1) + 2H(G_2)M_1(G_1) + 4e(G_1)RDD(G_2) + 2n_1RGut(G_2) + n_2(n_2 - 1)RGut(G_1) + 4n_1(n_1 - 1)(n_2 - 1)e(G_2) + 4H(G_1)e(G_2)^2]$$

GUTMAN INDEX OF WREATH PRODUCT OF COMPLETE GRAPHS

In this section, we find the reciprocal Gutman index of the wreath product $G_1°G_2$ of the complete graphs G_1 and $G_2$with odd vertices. Let $V(G_1) = \{u_0, u_1, \cdots, u_{n_1-1}\}, V(G_2) = \{v_0, v_1, \cdots, v_{n_2-1}\}$ and let w_{ij} be the vertex (u_i, v_j) of $G_1°G_2$.

The following lemma follows from the definition of the wreath product of the two graphs.

Lemma – 3.1

Let $G_1$and G_2 be two complete graphs. Let $w_{ij} = (u_i, v_j)$ and $w_{pq} = (u_p, v_q)$ be in $V(G_1°G_2)$. Then $d_{G_1\blacksquare G_2}(w_{ij}, w_{pq}) = \begin{cases} d_{G_1}(u_i, u_p), if\ i \neq p \\ 1, \quad if\ j \neq q \end{cases}$ and $d_{G_1°G_2}(w_{ij}) = |V(G_2)|d_{G_1}(u_i) + d_{G_2}(v_j)$.

Theorem – 3.2

If $G_1$and G_2 are two complete graphs with odd vertices$|V(G_1)| = n_1$and $|V(G_2)| = n_2$, where $n_1, n_2 \geq 3$ then

$$RGut(G_1°G_2) = n_2^2e(G_2)M_1(G_1) + 2n_2e(G_1)M_1(G_2) + n_1M_2(G_2) + n_2^3RGut(G1) + 2n_2RDD(G1)e(G_2) + H(G_1)M_1(G_2) + n_2^3(n_2 - 1)RGut(G_1) + 2n_2(n_2 - 1)e(G_2)RDD(G_1) + H(G_1)[4e(G_2)^2 - M_1(G_2)]$$

where $RGut(G), H(G), M_1(G), M_2(G)$ and $RDD(G)$ denote the reciprocal Gutman index, the Harary index , the first Zagreb index, the second Zagreb index and the reciprocal degree distance of G respectively.

Proof

Let $G = G_1°G_2$. Then

$$RGut(G) = \frac{1}{2}\sum_{w_{ij},w_{pq}\in V(G)} \frac{d_G(w_{ij})d_G(w_{pq})}{d_G(w_{ij},w_{pq})} =$$

$$\frac{1}{2}\left\{\sum_{j=0}^{n_2-1}\sum_{i,p=0,i\neq p}^{n_1-1} \frac{d_G(w_{ij})d_G(w_{pj})}{d_G(w_{ij},w_{pj})} + \sum_{i=0}^{n_1-1}\sum_{j,q=0,j\neq q}^{n_2-1} \frac{d_G(w_{ij})d_G(w_{iq})}{d_G(w_{ij},w_{iq})} + \sum_{j,q=0,j\neq q}^{n_2-1}\sum_{i,p=0,i\neq p}^{n_1-1} \frac{d_G(w_{ij})d_G(w_{pq})}{d_G(w_{ij},w_{pq})}\right\}$$

$$= \frac{1}{2}(A + B + C) \qquad (3.1)$$

where A, B, C are the sums of the above terms in order. We shall calculate A, B, C of (3.1) separately.
First we calculate,

$$A = \sum_{j=0}^{n_2-1}\sum_{i,p=0,i\neq p}^{n_1-1} \frac{d_G(w_{ij})d_G(w_{pj})}{d_G(w_{ij},w_{pj})}$$

For this we firsr compute

$$A' = \sum_{i,p=0,i\neq p}^{n_1-1} \frac{d_G(w_{ij})d_G(w_{pj})}{d_G(w_{ij},w_{pj})}$$

$$= \sum_{i,p=0,i\neq p}^{n_1-1} \frac{[n_2d_{G_1}(u_i)+d_{G_2}(v_j)][n_2d_{G_1}(u_p)+d_{G_2}(v_j)]}{d_{G_1}(u_i,u_p)}$$

by the Lemma – 3.1

$$= \sum_{i,p=0,i\neq p}^{n_1-1} \frac{n_2^2[d_{G_1}(u_i)d_{G_1}(u_p)]}{d_{G_1}(u_i,u_p)} + n_2\, d_{G_2}(v_j)\sum_{i,p=0,i\neq p}^{n_1-1} \frac{[d_{G_1}(u_i)+d_{G_1}(u_p)]}{d_{G_1}(u_i,u_p)} + d_{G_2}{}^2(v_j)\sum_{i,p=0,i\neq p}^{n_1-1} \frac{1}{d_{G_1}(u_i,u_p)}$$

$$= 2n_2^2RGut(G_1) + 2n_2d_{G_2}(v_j)RDD(G_1) + 2d_{G_2}^2(v_j)H(G_1)$$

by the definitions of reciprocal Gutman index, reciprocal degree distance and Harary index of a graph.

$$\therefore A = \sum_{j=0}^{n_2-1}\left[2n_2^2 RGut(G_1) + 2n_2 d_{G_2}(v_j) RDD(G_1) + 2d_{G_2}^2(v_j) H(G_1)\right]$$
$$= 2n_2^3 RGut(G_1) + 4n_2 e(G_2) RDD(G_1) + 2H(G_1)M_1(G_2) \quad (3.2)$$

Next we calculate

$$B = \sum_{i=0}^{n_1-1}\sum_{j,q=0,j\neq q}^{n_2-1}\frac{d_G(w_{ij})d_G(w_{iq})}{d_G(w_{ij},w_{iq})}$$

For this we compute

$$B' = \sum_{j,q=0,j\neq q}^{n_2-1}\frac{d_G(w_{ij})d_G(w_{iq})}{d_G(w_{ij},w_{iq})}$$
$$= \sum_{j,q=0,j\neq q}^{n_2-1}\left[n_2 d_{G_1}(u_i) + d_{G_2}(v_j)\right]\left[n_2 d_{G_1}(u_i) + d_{G_2}(v_q)\right] \quad \text{by lemma – 3.1}$$

$$= \sum_{j,q=0,j\neq q}^{n_2-1}\left[d_{G_2}(v_j)d_{G_2}(v_q)\right] + n_2\, d_{G_1}(u_i)\sum_{j,q=0,j\neq q}^{n_2-1}\left[d_{G_2}(v_j) + d_{G_2}(v_q)\right] + n_2^2 {d_{G_1}}^2(u_i)\sum_{j,q=0,j\neq q}^{n_2-1} 1$$

$$= 2\left[n_2^2 e(G_2) d_{G_1}^2(u_i) + n_2 d_{G_1}(u_i) M_1(G_2) + M_2(G_2)\right]$$

by the definitions of the first and second Zagreb indices of a graph.

$$\therefore B = \sum_{i=0}^{n_1-1} 2\left[n_2^2 e(G_2) d_{G_1}^2(u_i) + n_2 d_{G_1}(u_i) M_1(G_2) + M_2(G_2)\right]$$
$$= 2[n_2^2 e(G_2) M_1(G_1) + 2n_2 e(G_1) M_1(G_2) + n_1 M_2(G_2)] \quad (3.3)$$

Next we compute

$$C = \sum_{j,q=0,j\neq q}^{n_2-1}\sum_{i,p=0,i\neq p}^{n_1-1}\frac{d_G(w_{ij})d_G(w_{pq})}{d_G(w_{ij},w_{pq})}$$

For this we compute

$$C' = \sum_{j,q=0,j\neq q}^{n_2-1}\frac{d_G(w_{ij})d_G(w_{pq})}{d_G(w_{ij},w_{pq})}$$
$$= \sum_{j,q=0,j\neq q}^{n_2-1}\frac{[n_2 d_{G_1}(u_i) + d_{G_2}(v_j)][n_2 d_{G_1}(u_p) + d_{G_2}(v_q)]}{d_{G_1}(u_i,u_p)} \quad \text{by the Lemma – 3.1}$$

$$= \sum_{j,q=0,j\neq q}^{n_2-1}\frac{n_2^2[d_{G_1}(u_i)d_{G_1}(u_p)]}{d_{G_1}(u_i,u_p)} + n_2\sum_{j,q=0,j\neq q}^{n_2-1}\frac{[d_{G_1}(u_i)d_{G_2}(v_q) + d_{G_1}(u_p)d_{G_2}(v_j)]}{d_{G_1}(u_i,u_p)} + \sum_{j,q=0,j\neq q}^{n_2-1}\frac{d_{G_2}(v_j)d_{G_2}(v_q)}{d_{G_1}(u_i,u_p)}$$
$$= n_2^2 n_2(n_2-1)\frac{[d_{G_1}(u_i)d_{G_1}(u_p)]}{d_{G_1}(u_i,u_p)} + 2n_2(n_2-1)e(G_2)\frac{[d_{G_1}(u_i) + d_{G_1}(u_p)]}{d_{G_1}(u_i,u_p)}$$
$$+ (4e(G_2)^2 - M_1(G_2))\frac{1}{d_{G_1}(u_i,u_p)}$$

$$\therefore C = \sum_{i,p=0,i\neq p}^{n_1-1}\left[n_2^2 n_2(n_2-1)\frac{[d_{G_1}(u_i)d_{G_1}(u_p)]}{d_{G_1}(u_i,u_p)} + 2n_2(n_2-1)e(G_2)\frac{[d_{G_1}(u_i) + d_{G_1}(u_p)]}{d_{G_1}(u_i,u_p)} + (4e(G_2)^2 - M_1(G_2))\frac{1}{d_{G_1}(u_i,u_p)}\right]$$
$$= n_2^3(n_2-1)RGut(G_1) + 4n_2(n_2-1)e(G_2)RDD(G_1) + 2(4e(G_2)^2 - M_1(G_2))H(G_1) \quad 3.4$$

Using 3.2, 3.3, 3.4 in 3.1 we get

$$RGut(G_1{\circ}G_2) = n_2^2 e(G_2)M_1(G_1) + 2n_2 e(G_1)M_1(G_2) + n_1 M_2(G_2) + n_2^3 RGut(G1) + 2n_2 RDD(G1)e(G_2) + H(G_1)M_1(G_2) + n_2^3(n_2 - 1)RGut(G_1) + 2n_2(n_2 - 1)e(G_2)RDD(G_1) + H(G_1)[4e(G_2)^2 - M_1(G_2)]$$

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